INTRODUCTION TO SOME PROPERTIES OF GENERAL TOPOLOGY

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ABSTRACT

Topology is centered on the properties of a geometric objects that keep under continuous deformity, similar as twist, stretching, crumpling and bending, without closing and opening holes, tearing, or passing through itself. It presents a really thorough treatment of general ideas, similar as convergence and functions, which are frequently studied by using a topology. Currently one speaks further and further about the specialization in current science. Indeed though this statement is valid up to a certain point, one might say that the characteristic of science today is the every time greater intercourse among the varied disciplines that authenticated.Also to what happens in science in general, in each discipline one pursues a broader relationship among the different fields that confirm it. One of these mathematical generalizations is that of a topological space, that includes everything related with "nearness", "continuity", "neighborhood", "distortion", etc. The major emphasis is on weak topological properties similar to the separating axioms and covering properties. Outstanding features are the successful combination of lattice theoretic and filter-theoretic notions, and the content of the Stone- Cech compactification .Then I present some properties of general topology in this composition. General topology or Point set topology typically considers native properties of spaces and is nearly related to analysis. It generalizes the notion of continuity to define topological spaces, in which limits of sequences can be considered. The basic ideas in general topology are those of convergence and continuity. Although both have appeared in implicit forms

for a long time, it was only with the appearance of analysis that precise descriptions were given for these notions. This part was also isolated and came to be known as topology.

KEYWORDS : Topological spaces, continuity, connectedness, Euclidean spaces, Knot theory

INTRODUCTION

Topology is the branch of mathematics that studies continuous distortions of geometric objects. The word 'Topology' is derived from two Greek words, topos and *logos* therefore topology literally means the study of surfaces. One of the purposes of topology is to classify objects, or at least, to give approaches in order to distinguish between objects that aren't homeomorphism (a continuous one- to- one mapping between topological spaces that has a continuous inverse mapping). Namely, to decide between objects that cannot be gained from each other through a continuous distortion. Topology also provides manners to study topological structures in objects that arise in different fields of mathematics.

A circle and a square are topologically the same. Physically, a rubber band can be stretched into the form of a circle or a square, as well as multiple other shapes which are also viewed as being topologically the same. On the other hand, a figure eight arc formed by two circles touching at a point is to be regarded as topologically distinct from a circle or square. A qualitative property that distinguishes the circle from the figure eight is the number of connected pieces that remain when a single point is removed When a point is removed from a circle what remains is still connected, a single arc, whereas for a figure eight if one removes the point of contact of its two circles, what remains is two separate arcs, two separate pieces.

Topology can be divided into algebraic topology (which include combinatorial topology), differential topology, and low- dimensional topology. The low level language of topology, which isn't really considered a separate branch of topology, is known as point- set topology or general topology. General topology is useful for giving a rigorous treatment of multitudinous ideas in analysis and shape. Most of the time, one has an object one is interested in for some usage and one finds a natural way to induce a topology.

One of the fundamental problems of Topology is to determine when two given geometric objects are homeomorphisms. This can be somewhat tough in general. Spaces that can be mapped onto each other by a homeomorphism are regarded as the same in general topology. The term used to describe two geometric objects that are topologically the same is homeomorphism. Therefore a circle and a square are homeomorphisms. Primarily, if we place a circle C inside a square S with the same center point, also projecting the circle radially outward to the square defines a function $f: C \rightarrow S$, and this function is continuous small changes in x produce small changes in f (X). The function f has an inverse $f^{-1}: S \rightarrow C$ acquired by projecting the square radially inward to the circle and this is continuous as well. One says that f is a homeomorphism between C and S.

TOPOLOGICAL SPACES

A topological space is defined as a convex if a straight line joining any two points on the space is contained entirely in the space: probability space is commonly viewed as a convex space in which we allow only homeomorphic continuous transformations.

Topological spaces are the objects for which continuous functions can be defined. For the sake of simplicity and concreteness let us talk about functions $f:\mathbb{R}\rightarrow\mathbb{R}$. There are two definitions of continuity for such a function: the ε-δ definition and the definition in terms of limits. But it is a third definition, equivalent to these two, that is the one we want here. This definition is expressed in terms of the notion of an open set in R, generalizing the familiar idea of an open interval (a, b).

Definition:

A subset O of R is open if for each point X ∈ O there exists an interval (a, b) that contains x and is contained in O. With this description an open interval clearly qualifies as an open set. Other instances are that R itself is an open set, as are semi-infinite intervals (a, ∞) and (− ∞, a). The complement of a finite set in R is open. However, n = 1, 2, If A is the union of the unlimited sequence Xn = 1/n. Any union of open intervals is an open set. The antedating examples are special cases of this. The converse statement is also true every open set O is a union of open intervals since for each X ∈ O there's an open interval(a_X, b_X) with X ∈(a_X, b_X) ⊂ O, and O is the union of all these intervals(a_X, b_X). The empty set ∅ is open, since the condition for openness is satisfied vacuously as there are no points x where the condition could fail to hold.

Then there are some instances of sets which aren't open. A closed interval (a, b) isn't an open set since there's no open interval about either a or b that's contained in (a, b). Also, half-open intervals (a, b) and (a, b) aren't open sets when a< b. A nonempty finite set isn't open. Now for the nice description of a continuous function in terms of open sets

Definition:

A function f: R → R is continuous if for each open set O in R the inverse image f−1 (O) = X ∈ R f(X) ∈ O is also an open set. To see that this corresponds to the intuitive notion of continuity, consider what would be if this condition failed to hold for a function f. There would also be an open set O for which $f^{-1}(O)$ wasn't open. This means there would be a point $X_0 \in f^{-1}(O)$ for which there was no interval(a, b) containing X_0 and contained in $f^{-1}(O)$. This is an alternative to saying there would be points x arbitrarily close to X_0 that are in the complement of $f^{-1}(O)$. For X to be in the complement of $f^{-1}(O)$ means that f(X) isn't in O. On the other hand, X_0 was in $f^{-1}(O)$ so $f(X_0)$ is in O. Since O was assumed to be open, there's an interval(c, d) about $f(X_0)$ that's contained in O. The points f(X) that aren't in O are thus not in(c, d) so they remain at least a fixed positive distance from $f(X_0)$.

To abstract there are points X arbitrarily near to X_0 for which f(X) remains at least a fixed positive distance away from $f(X_0)$. This clearly says that f is discontinuous atX_0.This reason can be reversed. A reasonable interpretation of discontinuity of F at X_0 would be that there are points X arbitrarily close to X_0 for which f(X) stays at least a fixed positive distance away from $f(X_0)$. Call this fixed positive distance ε. Let O be the open set $(f(X_0) - \varepsilon, f(X_0) + \varepsilon)$. Also $f^{-1}(O)$ contains X_0 but it doesn't contain any points X for which f(X) isn't in O, and we're assuming there are similar points X arbitrarily close to X_0, so $f^{-1}(O)$ isn't open since it doesn't contain all points in some interval(a, b) about X_0.

The description we've given for continuity of functions $R \to R$ can be applied more generally to functions $Rn \to Rn$ and even $R_m \to Rn$ once one has a notion of what open sets in Rn are. The natural description generalizing the case n = 1 is to say that a set O in Rn is open if for each $X \in O$ there exists an open ball containing X and contained in O, where an open ball of radius r and center X_0 is defined to be the set of points X of distance less than r from X_0. Then, the distance from X to X_0 is measured as in linear algebra, as the length of the vector $X - X_0$,the square root of the dot product of this vector with itself. This description of open sets in Rn doesn't depend as heavily on the notion of distance in Rn as might appear.

For illustration in R_2 where open "balls" are open disks, we could use open squares rather of open disks since if a point $X \in O$ is contained in an open disk contained in O also it's also contained in an open square contained in the disk and hence in O, and again, if X is contained in an open square contained in O also it's contained in an open disk contained in the open square and hence in O. In such a way we could use numerous other shapes besides disks and squares, similar as ellipses or polygons with any number of sides. After these primary notes we now give the description of a topological space.

Definition:

A topological space is a set X together with a collection O of subsets of X, called open sets, such that:

(1) The union of any collection of sets in O is in O.

(2) The intersection of any finite collection of sets in O is in O.

(3) Both $\emptyset$ and X are in O. The collection O of open sets is called a topology on X.

OPEN AND CLOSED SETS

The union (or intersection) of finitely multiple open subsets is open. As its duality, the intersection (or union) of finitely multiple closed subsets is closed. Now what can we say about the union (or intersection) of an open subset and a closed subset of a topological space?

In the following theorem, roughly speaking, we prove that the intersection of a connected open set and a closed set is open if and only if the closed set includes the open set.

Theorem 1

Assume that F is a closed subset and G is a nonempty connected open subset of a topological space X. also, $F \cap G$ is a nonempty open subset of X if and only if $G \subset F$.

Proof

Suppose $F \cap G$ is a nonempty open subset of X but $G \not\subset F$, that is, $G \setminus F = G \cap X \setminus F$ is a nonempty open subset of X. Then the relation $G = (G \setminus F) \cup (F \cap G)$ would lead to the contradiction that *G* is the union of two nonempty disjoint open sets, that is, *G* is not connected. Thus if $F \cap G$ is a nonempty open subset of X, then $G \subset F$. The converse is obvious.

CONNECTED SPACES

The space of real numbers R has the property that any partition of R by two nonempty convex subsets S1 and S2 has a(unique) cut point x. The partition itself is called a Dedekind cut and the cut point is a point in the closure of both S1 and S2. The reality of cut points is traditionally described as an abstract of the completeness of R, but then we view it as related to the general topological phenomenon of connectedness.

Definition 1

A space X is connected if it cannot be broken down as the union of two disjoint nonempty open sets. This is equivalent to saying X cannot be broken down as the union of two disjoint nonempty closed sets. A third equivalent condition is that the only sets in X that are both open and closed are $\emptyset$ and X itself. For if A were any other set that was both open and closed, also X would be broke down as the union of the disjoint nonempty open sets A and $X - A$. Again, if X were the disjoint union of the nonempty open sets A and B also A would be closed as well as open, being equal to the complement of B, and A would be neither $\emptyset$ nor X.

Note

It turns out we can drop the convexity condition and replace it with the condition that S1 and S2 be nonempty; cut points will still be although we may lose the distinctiveness of the cut point if the sets aren't convex. The notion of connection point is a generality of the notion of cut point.

Definition 2

Suppose S1 and S2 are two subsets of a topological space X. A point $x \in X$ is called a connection point for S1 and S2 if $x \in S1$ and $x \in S2$. If $x \in S$ then we say that x is a contact point of the subset S(this includes all limit points of S together with any point of S that isn't a limit point of S). So a connection point for S1 and S2 is simply a point that's a contact point for both S1 and S2. We proved already that if x is a contact point of S in a topological space X, also the image f (X) is a contact point of f (S) in the space Y for any continuous function $f: X \to Y$. This implies that being a connection point is maintained by continuous functions as well.

Proposition 1

Suppose $f: X \to Y$ is continuous. If $x \in X$ is a connection point for subsets S1 and S2 also f (X) is a connection point for f (S1) and f (S2). Instinctively we view a contact point as supplying a “connection” or a “bridge” between two sets. This leads to the notion of connectedness.

Definition 3

A space X is said to be connected if, for all partitions of X by two nonempty subsets S1 and S2, there's a connection point for S1 and S2.

Proposition 2

Suppose f: X → Y is continuous and surjective. If X is connected also so is Y.

KNOT THEORY

Another branch of algebraic topology that's involved in the study of three-dimensional manifolds is knot theory, the study of the ways in which entangled replications of a circle can be embedded in three- dimensional space. Knot theory, which dates back to the late 19th century, gained increased attention in the last two decades of the 20th century when its implicit usages in physics, chemistry, and biomedical engineering were honored. Knot theory considers questions similar as the following;

Given a knotty loop of string, is it really entangled or can it, with enough imagination and/ or luck, be untangled without having to cut it? More generally, given two complicated loops of string, when are they deformable into each other? Is there an effective algorithm (or any algorithm to speak of) to make these determinations? Although there has been nearly explosive growth in the number of important results proved since the discovery of the Jones polynomial, there are still multiple" knotty" problems and conjectures whose answers remain unknown.

CONTINUITY

An important characteristic of general topological spaces is the ease of defining continuity of functions. A function f mapping a topological space X into a topological space Y is defined to be continuous if, for each open set V of Y, the subset of X conforming of all points p for which f (P) belongs to V is an open set of X. Another account of this description is easier to visualize. A function f from a topological space X to a topological space Y is continuous at p ∈ X if, for any neighborhood V of f (P), there exists a neighborhood U of p similar that f (U) ⊆ V.

These descriptions give important conceptions of the usual notion of continuity studied in analysis and also allow for a straightforward conception of the notion of homeomorphism to the case of general topological spaces. Therefore, for general topological spaces, unchanging properties are those conserved by homeomorphisms.

1. A Criterion for Continuity

We have to show that, if a topological space X is the union of a finite collection of closed sets, and if a function from X to some topological space is continuous on each of these closed sets also that function is continuous on X.

Lemma 1.a

Let X and Y are topological spaces, let f X → Y is a function from X to Y, and let X = A1 ∪ A2 ∪ • • • ∪ Ak, where A1, A2. A $_k$ are closed sets in X. Suppose that the restriction of F to the closed set Ai is continuous for i = 1, 2, k. also f: X → Y is continuous.

Proof

A function f X → Y is continuous if and only if f −1 (G) is closed in X for every closed set G in Y. Let G be a closed set in Y. also f −1 (G) ∩ Ai is relatively closed in Ai for i = 1, 2...k. Since the restriction of f to Ai is continuous for each i. But Ai is closed in X, and thus a subset of Ai is fairly closed in Ai if and only if it's closed in X. thus f −1(G) ∩ Ai is closed in X for i = 1, 2... k. Now f −1: (G) is the union of the sets f −1 (G) ∩ Ai for i = 1, 2... k. It follows that f −1 (G), being a finite union of closed sets, is itself closed in X. It now follows from another Lemma that f: X → Y is continuous. Illustration, Let Y be a topological space, and let α(0, 1) → Y and β(0,1) → Y be continuous functions defined on the interval(0, 1), where α(1) = β(0). Let γ(0,1) → Y be defined by γ(t) = α(2t) if 0 ≤ t ≤ 1 2; β(2t − 1) if 1 2 ≤ t ≤ 1. Now γ|(0, 1 2) = α ∘ ρ where ρ(0, 1 2) →(0, 1) is the continuous function defined by ρ(t) = 2t for all t ∈(0, 1 2). Therefore γ (0, 1 2) is continuous, being a composition of two continuous functions also γ (1 2, 1) is continuous. The subintervals(0, 1 2) and(1 2, 1) are closed in(0, 1), and(0, 1) is the union of these two subintervals. It follows from Lemma 1.a that γ (0, 1) → Y is continuous.

EUCLIDEAN SPACES

Euclidean n- space, occasionally called Cartesian space or simply n- space is the space of all n- tuples of real numbers, (x1, and x2… xn). Like n- tuples are occasionally called points, although other designation may be used. The whole of n- space is generally denoted Rn, although ancient literature uses the symbol En. Rn is a vector space and has Lebesgue covering dimension n. For this reason, essentials of Rn are occasionally called n-vectors.R_1 = R is the set of real numbers (i.e., the real line), and R_2 is called the Euclidean plane In Euclidean space, covariant and contra variant amounts are equivalent. In current mathematics, Euclidean spaces form the prototypes for other, more complicated geometric objects.

For illustration, a smooth manifold is a Hausdorff topological space that's locally diffeomorphic to Euclidean space. Diffeomorphism doesn't regard distance and angle, so these vital notions of Euclidean geometry are lost on a smooth manifold. Still, if one also prescribes an easily varying inner product on the manifold's digression spaces, also the result is what's called a Riemannian manifold. Put another way, a Riemannian manifold is a space constructed by distorting and repairing together Euclidean spaces. Such a space enjoys sundries of distance and angle, but they acquit in a twisted, non-Euclidean manner. The simplest Riemannian manifold, according to Rn with a constant inner product, is fundamentally identical to Euclidean n- space itself.

Still, also the result is a pseudo-Euclidean space, if one alters a Euclidean space so that its inner product becomes negative in one or another direction. Smooth manifolds made from similar spaces are called pseudo-Riemannian manifolds. May be their most well-known usage is the proposition of relativity, where empty space time with no matter is represented by the flat pseudo-Euclidean space called Minkowski space, space times with matter in them from other pseudo-Riemannian manifolds, and gravity corresponds to the angle of such a manifold.

Our world, being subject to reliance, isn't Euclidean. This becomes significant in theoretical considerations of astronomy and cosmology, and also in some practical problems similar to global positioning and aero plane navigation. Nevertheless, a Euclidean model of the world can still be used to answer numerous other practical problems with sufficient precision.

CONCLUSION

The goal of the study is to explore some properties of general topology. In this review paper, the property similar to connectedness and continuity of topology spaces was argued. Topology is the mathematical study of the properties that are conserved through distortion, twist, and stretching of objects. Tearing, still, isn't allowed. A circle is topologically equivalent to a sphere (into which it can be distorted by stretching) and a sphere is equivalent to an ellipsoid. Before studying topology you should clearly know real/ complex analysis, and you should also have some familiarity with abstract algebra.Geometry and topology is constantly treated as an intersection of analysis and algebra. It's also used in string theory in physics, and for describing the space- time structure of the world. Topology has been used to study various biological systems including atoms and nanostructure (e.g., membranous objects). In particular, circuit topology and knot theory have been considerably applied to classify and compare the topology of folded proteins and nucleic acids.

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